Errata
To *Tangent Graeffe Iteration*, by Gregorio Malajovich and Jorge P. Zubelli
*Numerische Mathematik* 89 749–782 (2001), DOI 10.1007/s002110100278

## 1 Known typos

p. 756 In Algorithm 1, the second line from the bottom should read:

\[ t \leftarrow \alpha_2 + \alpha_1 e^{h \Delta} \]

p. 761 Line 10 should read:

\[- \log |f_0| + \sum_{j \leq i} \log |\zeta_j|\]

On line 12, we should also refer to the convex hull of \( i \mapsto - \log |f_0| + \sum_{j \leq i} \log |\zeta_j| \).

p. 763 The \( i \) should be replaced by \( d - l \).

p. 764 In equations (4) and (5), \( c \) should be replaced by different variables \( c_1 \) and \( c_2 \), with \( |c_1|, |c_2| < 2d R^{-1} \ll 1 \). Then, by the Intermediate Value Theorem, there is \( c'' \) such that \( \log |1 + c_1| - \log |1 + c_2| = 2 \log |1 + c''| \).

p. 765 In lines 7 and 10, the error term should be divided by \( i_2 - l \) and \( l - i_1 \), respectively. This does not affect the statement of Lemma 3.

p. 769 In Algorithm 3, all powers of \((-1)^d\) should be multiplied by \((-1)^d\). In the last line, writing \((-1)^{d+i+j}\) would look nicer than \((-1)^{d+i+j}\).

p. 771 Lemma numbering is wrong. Algorithm 4 refers to Lemma 6 and Algorithm 5 refers to Lemma 5.

p. 773 The definition of \( a \) and \( b \) is wrong. It refers to a previous version of the algorithm, where the definition was:

\[ a = \frac{\dot{g}_{i_k+d}}{g_{i_k+d}} \quad b = \frac{\dot{g}_{i_k}}{g_{i_k}} \]

Mike McNamee pointed out that \( a \) was subtracted from \( b \) in Algorithms 4 and 5, so the correct statement is, at the end of page 772:

\[ \frac{2^{-N}}{d^r} \tilde{b} = \frac{\zeta_{i_k+1}}{|\zeta_{i_k+1}|^2} (1 + \delta_2) \]

and in the first line of page 773, we mean: where \( b \) is as in the Algorithms.
2 Further clarifications

**Question:** In page 763 line -7, why are the terms in $\Sigma$ smaller than $R^{-1}|Z_{i_2}|^{i_2-1}|Z_{i_2+1}| \ldots |Z_d|$?

The reason is that the terms in $\Sigma'$ have all modulus

$$|Z_{i_2}|^{i_2-1}|Z_{i_2+1}| \ldots |Z_d|$$

The terms in $\Sigma$ have at least one of the factors replaced by a smaller root $|Z_j| < R^{-1}Z_{i_2}$. Hence the bound.

**Question** Can you explain the comment (p. 772) about decreasing $\rho$ at the end of Algorithm 6?

Algorithm 6 is a theoretical iterative algorithm. It produces a sequence of answers $(\zeta_N^N)_{N \in \mathbb{N}}$, and it is convergent.

This setting is necessary if we want the algorithm to return the “right answer” for all inputs. Otherwise, we could have a stopping criterion, after assuming a bound for the root circle separation $\rho$. This is possible for a circle-free polynomial with integer coefficients (Theorem 2.7 in Malajovich, Condition Number Bounds For Problems with Integer Coefficients, *Journal of Complexity* 16, 529–551 (2000)). This is also possible in a probabilistic setting, with a positive error probability (I have no estimate available).

But our result is stronger, in the sense that we prove convergence for all circle-free polynomials. The trick is to let $\rho$ converge to one at a convenient speed.

3 Thanks

We thank Mike McNamee for pointing out most of the typos above. Thanks to Caio Souza for some of them.