Lower Bound Techniques for Parallel Algorithms

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There have been significant advances in lower bound techniques for finding solutions to weighted combinatorial problems in parallel. This survey talk will attempt to give an outline of this basic technique of Mulmuley that uses real algebraic geometry to reduce the problem of finding lower bounds to the construction of explicit combinatorial objects.

This technique is a significant advance on the technique developed by Ben-Or and Yao which was first used to give lower bounds for elementary combinatorial problems.

The model of computation we consider is the Parallel RAM (PRAM) without bit operations. The model eliminates those operations that allow bit-extraction or updates of the bits of the individual registers, but provides the usual arithmetic, indirect referencing, conditional and unconditional branch operations at unit cost. We consider here an unbounded fan-in model, in which the operations \{+, \text{min}, \text{max}\} can have unbounded fan-in at unit cost.

This model of computation is natural, and virtually all known parallel algorithms for weighted optimization and algebraic problems fit into this model.

This talk will try and provide the intuition behind Mulmuley’s technique, and then try and cover the general ideas behind the combinatorial constructions used to provide lower bounds for problems like MAX-FLOW, WEIGHTED SHORTEST PATH, and extensions to some of these.