Can We Optimize Toeplitz/Hankel Computations?

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The classical and intensively studied problem of solving a Toeplitz/Hankel linear system of equations is omnipresent in computations in sciences, engineering and communication. Its equivalent formulations include computing polynomial gcd and lcm, Padé approximation, and Berlekamp-Massey’s problem of recovering the linear recurrence coefficients. To improve the current fastest divide-and-conquer algorithm by Morf 1974/1980 and Bitmead and Anderson 1980, we rely on Hensel’s $p$-adic lifting. We accelerate its recovery stage by exploiting randomization and the correlation between lifting and the computation of Smith’s invariant factors of the input matrix. Furthermore, for the average input, the 2-adic version of lifting is sufficient, allowing computations in binary form. Our resulting algorithms solve a nonsingular Toeplitz/Hankel linear system of $n$ equations by using $O(m(n)\mu(\log n))$ bit operations (versus the information lower bound of the order of $n^2 \log n$), where $m(n)$ and $\mu(d)$ bound the arithmetic and Boolean cost of multiplying polynomials of degree $n$ and integers modulo $2^d + 1$, respectively, and where the input coefficients are in $n^{O(1)}$. Our algorithms can be extended to solving nonsingular and consistent singular Toeplitz/Hankel-like linear systems of equations and computing the resultant, gcd and lcm of two polynomials of bounded degrees as well as a fixed entry of a Padé approximation table and the linear recurrence coefficients provided that the input values are some bounded integers.